Analysis of Composite Bridges in Practice – a Holistic Approach

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Summary

Composite structures are often used in bridge engineering. In most cases steel girders are combined with a cast in place concrete slab, but composite systems with precast concrete girders are also widely built. Although often applied in everyday structures, composite structures require challenging design procedures due to the necessity of considering the difference in behaviour of the materials and providing full connection between the structural parts.

These requirements are related to topics like properly considering the actual construction schedule in order to assign different loading components to the respective structural system. Further special needs arise in the simulation of certain loadings such as self weight of wet concrete or temperature differences between the different materials. A third major problem is related to correct simulation of long time behaviour redistributing stresses from the concrete slab to the girders and to dealing with pre-stressing in case of structures with pre-stressed concrete girders.

Once deformations and internal forces have been correctly calculated there still remain several problems relating to result assessment and proof checks. Stress and strain distribution over the composite section is no longer linear, i.e. stress checks have usually to be done separately for the individual parts. Ultimate load checks however relate to the final composite section. Proof checks for the shear keys between the different parts complete the set of requirements.

Consistent analysis procedures considering all these special problems have recently been developed and are presented herein. The solution allows for considering all required effects with sufficient accuracy in comprehensive manner and in accordance with generally accepted theories.

Keywords: composite structure, non-linear stress distribution, non-linear temperature distribution, creep and shrinkage, long-term effects, shear keys

1. Basics

1.1 Introduction

Structural analyses for bridges are usually performed with beam models allowing for performing all required proof checks in accordance with the respective valid design code. Basic assumption of this approach is that we have a purely linear stress and strain state in the beam cross-section, what is not the case anymore in composite beams.

The solution described in this paper is working with an extended beam approach [1] based on establishing a full geometric model of the superstructure cross-section with assigning the actual material parameters and activation time to the individual parts. Different sets of structural elements are defined, modelling on the one hand the individual parts and on the other hand the composite section.

1.2 Composite Cross-sections

The composite cross-sections consist of different *cross-section parts*. The individual parts are parts with different material and/or parts being activated at different points in time in the schedule. The geometry definition (FE-mesh) is done for the whole cross-section with the different parts consistently connected to each other.

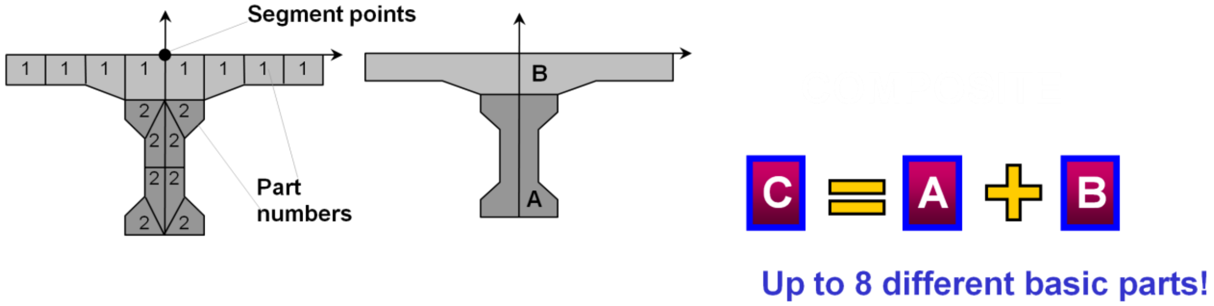


Figure 1 Composite cross-section with cross-section parts

For composite cross-sections consisting of parts with different material, the geometry information is not sufficient for calculating the correct cross-section values: the different parts have to be weighted in accordance with their stiffness parameters (Young's modulus for bending and normal force terms, shear modulus for shear terms). The calculated cross-section values are related to the parameters of the material assigned to the respective structural elements. The weighting factors used for the different cross-section parts are the ratios between the moduli of the actual cross-section part and the structural element.

1.3 Composite Elements

The basic concept for the analysis is such, that *partial elements* are combined to *composite elements*. Each part of the composite element (steel girder, concrete plate, etc) is defined as a separate partial element with the respective individual *cross-section part(s)* assigned at the start and end points. A separate composite element has to be defined for any – intermediate or final - combination of partial elements active at least once in the schedule.

Partial elements and composite elements are usually related to the same system line, i.e. they are allocated to the same series of nodal points and thus fully connected to each other. The hierarchical structure of the computation model allows for considering at any time the correct connectivity between partial and composite elements.

2. Construction schedule

2.1 General

Bridge structures are almost always built in stages, where the different construction parts are erected at different times and the related erection loading is acting on different intermediate structural systems. The final stressing state is generally very much dependent on the erection sequence. This particularly applies to composite structures, where not only additional structural parts (e.g. further spans) are assembled during erection, but already active structural members are successively completed. Using a 4D approach where the time domain is taken properly into account is therefore essential. The erection sequence is thereby described in the (construction) *schedule*.

The schedule contains different *stages*, where the currently active system is specified (new elements may be activated or de-activated), and the corresponding loading is applied. The stages are arranged along a global time axis, which in general mirrors the actual construction time of the bridge. The time scale itself governs the creep and shrinkage behaviour, i.e. if no creep and shrinkage occurs (e.g. steel bridges) then only the sequence of activation and load application is relevant.

2.2 Activation of elements

Each *stage* contains the information on elements, activated or de-activated at the beginning of it, and *schedule actions* applied within this stage. These actions are *calculation actions*, e.g. for calculating the stress state due to permanent loading or any intermediate live loading, but also *checking actions* and *list/plot actions* for evaluating design code checks or performing the result representation for the respective stage. The time scale is defined by *creep actions*, which define the application time of the different actions.

Each stage can be divided into more than one creep interval, but activation and de-activation of elements is done at the beginning of a stage, i.e. each stage represents a certain “active structural system”. For composite structures this includes defining a certain composition state of the active elements.

2.1 Establishing the composite state in the schedule

The partial elements and composite elements may be activated in the different construction stages of the schedule in accordance with the sequence they are supplemented on site. Note that in any composite stage the *composite elements* representing the current combination of *partial elements* and all belonging partial elements are active. The program automatically detects all elements being part of an active composite element, and omits them in the stiffness calculation process.

Stage 1	Stage 2	Stage 3
101-120 Part1	201-220 Part2	301-320 Part3
1001 Support	401-420 Composite(4)	501-520 Composite(5)
		De-Activate 401- 420

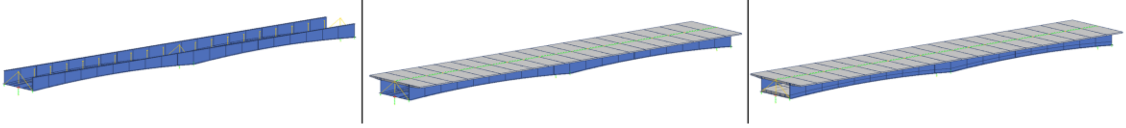


Figure 2 Example for successive activation of individual and composite elements

3. Special loading conditions – self weight, pre-stressing, temperature

3.1 Self weight – body loads

Body loads are generally automatically calculated by using the element length, cross-section area and specific weight of the material in order to minimize required user input. Two problems arise in this context for composite structures:

- Specific weight is usually not available for the composite cross-section and calculating fictitious specific weight values for the composite element may be a tedious process.
- Wet concrete weight acts when the respective partial elements are not yet active.

The approach with separate sets of elements for representing the individual parts and the composite members allows for handling these problems consistently: body loads can be related to the partial elements with realistic material properties. The calculated loading is automatically applied in the correct position on the related composite element. Wet concrete weight can also be easily defined with allowing the application of self-weight loads on inactive partial elements.

3.2 Temperature

Considering the *temperature impact*, we must differentiate between a linear and a non-linear temperature distribution over the cross-section.

In the case of constant or linear temperature distribution we must take into account that this impact – although itself linear – can cause a non-linear stress state in the composite cross-section if the temperature expansion coefficients of the materials are different. Fortunately expansion coefficients of steel and concrete can generally be assumed identical, so that this effect can mostly be neglected.

Considering non-linear temperature distribution is required in different design codes. AASHTO, British and Australian codes have for instance general prescriptions for temperature diagrams with temperature differences at top and bottom of the section decreasing to zero over a certain depth top down or bottom up. The edge value at cross-section top is usually higher than that at bottom, however dependent on the thickness of the roadbed or ballast. Eurocode also requires taking into account a distinct temperature difference between steel and concrete part.

The program allows for defining any temperature diagram as a function of depth below the cross-section surface. The expansion coefficient is taken from the assigned material. Any expansion differences between the materials can be taken into account by appropriately weighting the temperature diagram values. The respective diagram can be separated in an equivalent linear part and a primary part. The linear part is used in the analysis for calculating the secondary internal forces. The primary part is superimposed in the calculation of the stress distribution in the cross-section.

3.3 Pre-stressing of Composite Girders

Pre-stressing is related to structures where precast concrete girders are combined with a cast in place slab. One possibility is that the individual girders have been pre-stressed with straight strands in the manufacturing process. Another possibility is that the girders contain ducts. The tendons are placed on site and post-tensioned after the composite state is active. There is also the possibility that the tendons are partly tensioned before the slab is cast.

In our approach the tendons are always geometrically assigned to the partial elements representing the cross-section part where the tendons or ducts are located. All partial elements of a composite element may be individually pre-stressed. The pre-stressing is applied to the partial element where the tendon is situated. If the stressing actions are performed when the composite state is already active (or an intermediate composite state), the respective impact (primary pre-stressing state) is automatically applied on the respective active composite element.

“Primary” and “secondary” results must be distinguished and separately calculated and stored in order to allow for proper proof checking. The primary state contains direct effects of the tensioning process onto the pre-stressed structural elements (internal stress state without taking into account external constraints). The pre-stressing force is transformed into components N_x, Q_y, Q_z acting in axis direction perpendicular to it. The eccentric position (e_y and e_z) of the tendon with respect to the centre of gravity results the moments $M_x = Q_y * e_z + Q_z * e_y$, $M_y = N_x * e_z$, $M_z = N_x * e_y$.

These internal forces of the primary state are related to deformations of the pre-stressed elements, yielding deformations of the total system. External constraints (boundary conditions) will yield restraint stress resultants in the general case. The secondary state describes the internal forces due to restraint and the deformations of the total system within the boundary conditions.

4. Creep and shrinkage

4.1 General

Long-term effects due to creep and shrinkage of the concrete together with the specific structural behaviour make it imperative that appropriate techniques be used in the design and analysis process for considering these effects. The process must take into account all types of quasi-permanent loading and the time, when it is applied on the structure. This includes the computation of the effects due to creep and shrinkage in the time intervals between activating new structural components and

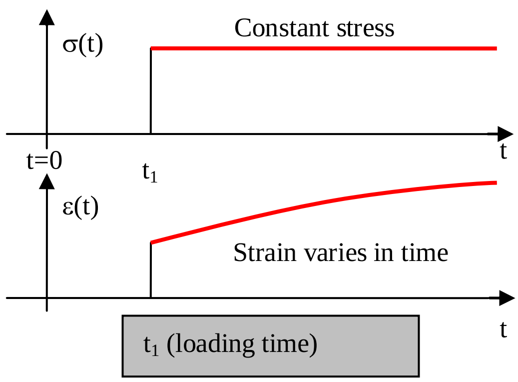


Fig. 3. Creep – development of the creep strain over the time under the constant stress

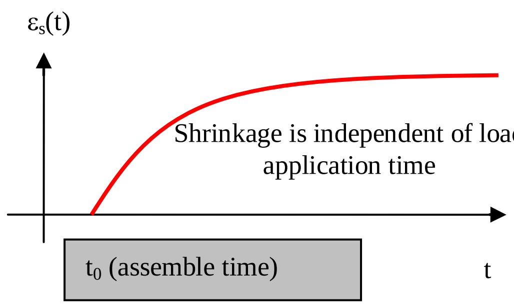


Fig. 4. Shrinkage – development of the shrinkage strain over the time

applying major new loadings.

The occurrence of time dependant plastic strain is a material property of concrete. It consists of two components, creep and shrinkage. Creep is a stress dependent material nonlinearity in which the material continues to deform under a constant load. Shrinkage does not depend on the load, and even an unloaded element will shrink.

The rules to determine the creep factor and shrinkage strain are very complex. Nowadays the CEB-FIP model is widely used. Many new design codes (EUROCODE, DIN, etc.) are based on these rules, with only minor differences to the original. Details are given in the CEB-FIP document [2].

4.1 Creep and shrinkage of composite structures

In composite structures – typically steel girders with concrete slabs – two types of material are combined within the deck section. Concrete layers with different ages act as different materials as they creep differently. Due to the difference of the creep capacity of the different parts (less creep of older concrete, no creep of steel), creep and shrinkage of concrete causes additional time-dependent strain and stress within the steel or precast concrete members.

The solution of the creep and shrinkage problem becomes more complicated because primary effects arise in addition as shown in *Fig 5*. The total stress in the composite section can be seen as the sum of primary and secondary stresses [3], [4], where the primary part represents the stresses due to the non-linear strain distribution in the cross-section plane, which are in equilibrium within the cross-section.

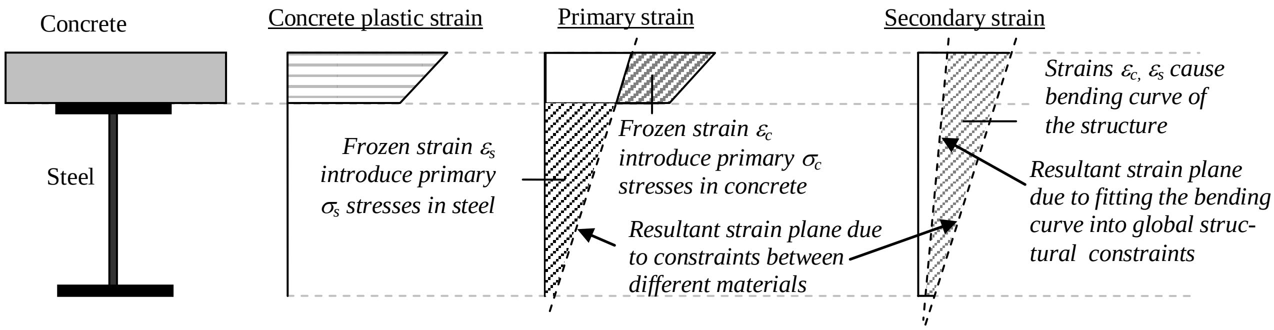


Fig. 5: Stress, creep factor and strain development in the time interval t_n to t_{n+1}

Applying the equilibrium condition on cross-section level leads to the actual strain plane resulting from constraints between different materials or different creep and shrinkage behaviour. The equations below show the equilibrium conditions for the primary stresses. The three components refer to longitudinal strains and gradients in y and z direction.

$$\sum N_x \equiv \int_{(A_c)} \sigma \cdot dA_c + \int_{(A_s)} \sigma \cdot dA_s = 0$$

$$\sum M_y \equiv \int_{(A_c)} \sigma \cdot z \cdot dA_c + \int_{(A_s)} \sigma \cdot z \cdot dA_s = 0$$

$$\sum M_z \equiv \int_{(A_c)} \sigma \cdot y \cdot dA_c + \int_{(A_s)} \sigma \cdot y \cdot dA_s = 0$$

Integrating the secondary strains along the girder length yields the theoretical bending curve of a statically determinate structure. Fitting the structure into external constraints produces additional strains and stresses in the composite girder.

4.1 Creep analysis – time stepping scheme

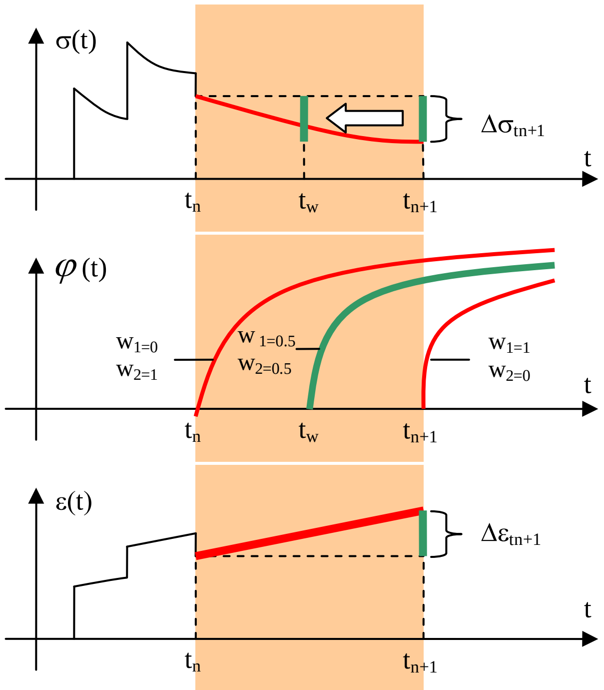
As creep is a stress dependent process, stresses arising in the concrete – both primary and secondary – essentially influence the creep rate. I.e. integration over the creep period is not a straight-forward process but requires iteration techniques such as time stepping with using an approximation within the individual time steps. The solution to the basic differential equation of the problem can now be written for the investigated creep interval [t_n, t_{n+1}] as follows:

$$\varepsilon_{t_{n+1}} = \varepsilon_{t_n} + \int_{\tau=0}^{t_n} \frac{1}{E} \cdot \frac{\partial \sigma_c}{\partial \tau} \cdot \left[\int_{t=t_n}^{t_{n+1}} \frac{\partial \varphi}{\partial t}(t,\tau,\gamma) \cdot dt \right] \cdot d\tau + \int_{t=t_n}^{t_{n+1}} \frac{\partial \varepsilon_s}{\partial t}(t,\gamma) \cdot dt + \int_{\tau=t_n}^{t_{n+1}} \frac{1}{E} \cdot \frac{\partial \sigma}{\partial \tau} \cdot \left[1 + \int_{t=\tau}^{t_{n+1}} \frac{\partial \varphi}{\partial t}(t,\tau,\gamma) \cdot dt \right] \cdot d\tau$$

$$\sigma_{t_{n+1}} = \sigma_{t_n} + \int_{t_n}^{t_{n+1}} \frac{\partial \sigma_c}{\partial t} \cdot dt$$

Simple cases, where structural and material linearity can be assumed and total permanent loading is applied at one specified point in time, also allow for a direct solution with adjusted stiffness, using an "*Age adjusted effective modulus*" $E_{adj} = E / (1+\varphi)$ where φ is the creep coefficient for the respective loading time.

However, a universally applicable algorithm must use an explicit or implicit time stepping scheme with using weighting factors w_1 and w_2 controlling the stress development within one time step. For $w_1 = 0$, the solution degenerates to the explicit time integration (forward Euler method). The strain rate at begin of the time interval is assumed constant in the whole interval Δt. Very small time steps are in this case required to minimize the error. The explicit approach is therefore not recommended for creep analysis.



$$t_w = t_n * w_2 + t_{n+1} * w_1$$

$$w_1 + w_2 = 1$$

t_w is effective load application time for stress increment arising during the creep interval.

Total stress increment at the end of creep interval is put "back" in time to cover "creep of creep" effect. This novel approach allows for consistent storage of creep results as "normal loading case" result.

Elastic strain is defined as given in *Eq. (6)*:

$$\varepsilon^e(t) = \varepsilon^e_{t_n} \cdot (1 - \frac{t - t_n}{\Delta t}) + (\varepsilon^e_{t_n} + \Delta\varepsilon^e_{t_{n+1}}) \cdot \frac{t - t_n}{\Delta t}$$

$$\varepsilon^e(t) = \varepsilon^e_{t_n} \cdot w_2 + (\varepsilon^e_{t_n} + \Delta\varepsilon^e_{t_{n+1}}) \cdot w_1$$

Fig. 6 Stress, creep factor and strain development in the time interval t_n to t_{n+1}

The method becomes implicit for all weighting factors with $w_1 > 0$, and $w1 = 1$ refers to backward Euler integration method. Other proposals for implicit time integration schemes are in literature [5], e.g. the central difference scheme ($w_1 = w_2 = 0.5$) or the Galerkin scheme ($w_1 = 2/3$ and $w_2 = 1/3$). Schemes with $w1 \geq 0.5$ are numerically unconditionally stable, i.e. they do not require very small time-steps. Implicit creep is generally more accurate, but accuracy is still dependent on the time-step length. A reasonable time-step must be used to capture the nonlinear creep behaviour accurately.

All implicit schemes require an iterative solution. Newton-Raphson iteration turned out to be appropriate for creep calculations and used in our approach [6].

5. Assessment of results and proof checks

5.1 Calculation of Internal Forces

Internal forces calculated for a certain load case are always related to the currently active composite cross-section. This implies problems in superimposing load case results calculated in different states. Special techniques must be applied to overcome this problem and get reasonable results for superposition of different load cases. In our approach we can select between 3 result options. Result option “*Normal*“ shows the internal forces as they are calculated. I.e. results of different states are not superimposed and assessment must always consider contributions of the composite elements as well as those of the corresponding partial elements.

The option “*Split*” may be used for splitting the results of composite elements to the belonging partial elements. In this case, the results shown for the composite elements remain unchanged, and the corresponding internal force components of the individual partial elements are additionally displayed. This splitting is done by using the well known standard formulas. The split results can be properly superimposed to previous internal forces of the partial elements.

The inverse option “*Joined*” allows for transforming normal results of partial elements to the element axis and cross-section of a later activated composite element. Although being a fictitious equivalent not describing the actual stressing state, these transformed internal force values may then be superimposed with the results of the composite elements in order to be used in ultimate load checks where only an overall equilibrium is considered.

5.2 Computation of Stresses

Proper stress calculation can only be done for the partial elements as strain distribution over the composite cross-section is not linear and respective material values can also be different. Stress calculation is therefore based on the superimposed “split” results of the partial elements as described above. Interesting stress points are defined at the very beginning for the different cross-section parts and stresses in these points are then automatically calculated and displayed for all these points.

5.3 Ultimate load check

Ultimate load check for composite structures is a delicate point because there are no clear rules in design codes how to deal with the non-linear strain state in the cross-section. A reasonable approach for the ultimate state is to assume stress redistribution so that the primary part disappears. This allows for using joined results for performing the ultimate load check in the common manner. I.e. a linear strain distribution is assumed and actual joined internal forces are increased until the ultimate strain state is reached.

6. Shear key design

6.1 Computation of Shear Key Forces

Shear keys are usually provided to shear deformation between the 2 parts of the composite section. Typical types are shown in Figure 7. Design forces in the shear keys must be determined in the design procedure in order to determine the required number and strength of the individual keys.

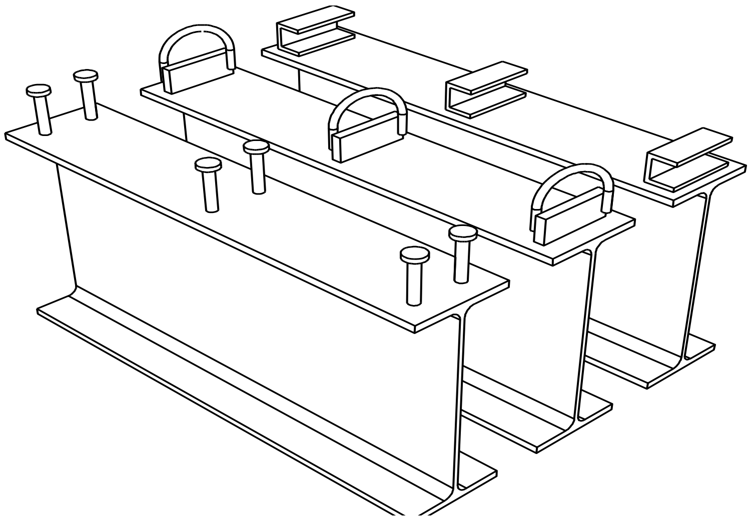


Figure 7 Different types of shear keys

For determining the necessary shear key dimensions, most design codes require calculating the shear forces in the connection face for the „ultimate serviceability state“. The following procedure is generally applied in traditional „by hand“ calculations: a horizontal section with the width „b“ is placed in the cross-section at the level of the connection face. The static moment „Sz“ of the cross-section part cut away is calculated. The shear stress may then be calculated using the well-known formula

$$\tau_{xy} = \frac{Q_y * S_z}{J_z * b}$$

However, the validity of this formula is limited (connection face parallel to the element axis, constant cross-section, etc.).

Therefore we use a more general approach: the shear stresses in the connection face must correspond to the change of the normal force (dN/dx) transmitted in that part of the composite element, which is separated by the considered connection face. These normal forces are available for all partial elements if the above described “*Split*” function has been used. Only the normal force difference between the start and the end of a partial element has to be calculated to get the total shear force being transmitted by the keys over the element length.

This process avoids the restrictions of the above shear stress formula and the result is consistent to all other system modelling assumptions. Special combination elements are defined in the database to allow for storing the shear key results like any other element results and for using them in all subsequent superposition and proof checking functions. In SLS design the required amount of shear keys can be easily determined by comparing the shear key forces with the allowable value of a single key.

6.1 Ultimate load check for shear keys

The design codes of several countries also claim an ultimate load check in addition to the described shear key calculation for the ultimate serviceability state. The German code (DIN) requires for instance, that the „plastic moment“ of the composite cross-sections has to be calculated and the related „total compression forces“ in the concrete part - and the related tension forces in the reinforcement of the cracked zone respectively - be determined.

The maximum value of these compression and tension forces is determined for different sections of the girder (mid-span region, support region) and the chosen shear key amount must be able to transmit these forces. However, these maximum forces may be reduced using the ratio between the actual „ultimate load moment“ (moment times safety factor) and the „plastic moment“. This reduction factor must not be less than 0.5.

In our approach the standard ultimate load check gives the ultimate moment of the composite cross-section. The „plastic moment“ of the composite cross-section is calculated by performing the ultimate moment calculation for a zero internal force combination. This is created by initialising an empty load case and assigning it to the ultimate moment calculation action without superimposing any calculated load cases.

7. Conclusion

The presented approach has been implemented in a commercially available computer program and successfully used in several practical applications. With using this holistic approach the involved structural engineers were able to accurately predict and follow the behaviour of composite bridges built in stages.

Following in detail the construction sequence with applying different loads in each case on the correct structural system and using sophisticated creep and shrinkage calculations allow for predicting deformations very accurately and defining required pre-camber shapes.

Using parallel sets of elements representing the composite members as well as the individual parts and the shear keys allow for performing the required proof checks in accordance with many different design codes. SLS checks (stress evaluation) are performed on the level of partial elements, ULS checks are related to internal force equilibrium in the composite section.

References

[1] Bentley Systems Austria, *RmBridge V8i Analysis User Guide*, Graz 2008

[2] *CEB-FIB Model Code,* Beuth Verlag**,** Berlin: 1990,1994

[3] Sattler, K., *Theorie der Verbundkonstruktionen*, Verlag Wilhelm Ernst & Sohn, Berlin, 1959.

[4] Peternelj, M., *Rheology of Steel-Concrete Composite Bridges*, Diploma Work, University of Maribor, April 2009.

[5] Zienkiewizc O.C., Taylor R.L., *The finite Element Method*, Fourth Edition, Volume 2, McGraw-Hill, 1994

[6] Janjic D., Stampler J., Bokan H., *Consistent Analysis Method for Long Term Effects in Composite Bridges*, Eurosteel Conference, Graz 2008