Consistent Numerical Model for Wind Buffeting Analysis of Long-Span Bridges

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Summary

The buffeting analysis of bridge structures considers both, the aero-elastic behaviour of the structure and the wind loading correlation. This calculation is commonly performed in the modal space and in the frequency domain. It includes aerodynamic damping and stiffness effects due to the structural movement caused by the wind flow. The aero-elastic parameters of the cross-sections (drag, lift, pitching moment and derivatives, usually determined in wind tunnel tests) are the basis of this type of analysis. The non-linear dependency of these parameters on the wind direction must be accounted for during both, the static mean-wind and the buffeting analysis.

The wind profile is characterised by the mean wind velocity and the fluctuation (turbulence) velocity where the height profile of the wind velocity is assumed to follow an exponential expression. The stochastic nature of wind loading (longitudinal, vertical and lateral components of the fluctuation velocity) is accounted for by using power spectra in the frequency domain. The spatial distribution (coherence) of the wind loading is commonly determined from an exponential decay function of the frequency and the geometry of the structure.

This paper reports on performing a buffeting analysis in a commercial bridge design computer program. The implementation of the buffeting analysis in a bridge design program means that all non-linear effects that may have taken place prior to the wind event can be included in the calculation. This allows a complete global analysis of bridge structures and has been applied to the buffeting analysis of the Rach Mieu Bridge in Vietnam and the Shenzhen Western Corridor as well as the Stonecutters Bridge in Hong Kong that serve as application examples in this paper.

Keywords: buffeting analysis, aero-elastics, dynamics, computer analysis, bridge design

1. Introduction

As dynamic effects due to wind loading, especially buffeting, often influence the design of long-span bridges considerably, the capability of performing a buffeting analysis was implemented in a commercially available bridge-design software package called *RM2004* (TDV 2004). This feature is now fully integrated with all the other functions of the software package.

Much effort has recently been devoted to further improve the dynamic functionality in *RM2004*. In addition to the previously available modules that included all standard procedures for earthquake design (eigenmode analysis, response spectrum analysis and time history based on modal analysis) a variety of new features have been added. The ability to perform buffeting analyses is among them.

Spectral definition of the wind loading requires that the analysis must include all frequency ranges (from zero to frequency infinity). Both, the structural and the aerodynamic damping must be considered in the analysis.

The structural buffeting calculation can be performed in the frequency domain (modal space) or in the time domain (direct integration in time). The stochastic nature of the wind and the spectral definition of the wind loading clearly indicate that a solution in the frequency domain is more appropriate. The analysis covers all frequency ranges with a consistent definition of the structural damping and aerodynamic damping and stiffness.

Structural and/or aerodynamic nonlinearities are generally required to be included in the calculation of longer-span bridges and therefore time domain approaches become more attractive again. Structural nonlinearities can be covered in the time domain, but several difficulties arise: the time history of the wind loading is unknown, the definition of the structural damping is restricted, and the numerical cut-off of the high-frequency response depends on the chosen time steps.

This paper reports on the implementation of a solution to the buffeting issue in the bridge design software package RM2004 [1]. This implementation is carried out in the frequency domain.

2. Buffeting Analysis – General Description

The structural buffeting calculation is performed in the modal space and in the frequency domain. It includes aerodynamic damping and stiffness effects due to structural movement caused by the wind flow. All computations are based on the tangential stiffness of the structure at a given moment in time – the structure under permanent loading and mean wind - ensuring the inclusion of all non-linear effects which may have taken place prior to the stochastic wind event.

The basis of the implemented buffeting analysis functions is the aero-elastic cross-section parameters of the structural members (drag, lift, pitching moment and derivations) as determined in wind tunnel tests. The dependency of these parameters on the wind direction is automatically accounted for during the static mean-wind analysis and the buffeting analysis in *RM2004*.

The wind profile is characterised by the mean wind velocity and the fluctuation (turbulence) velocity where the height profile of the wind velocity is assumed to follow an exponential expression. The stochastic nature of wind loading (longitudinal, vertical and lateral components) is accounted for by using power spectra in the frequency domain (Kaimal, Karman, Harris, Davenport or individually defined). Narrow band analysis as well as broad band analysis with full modal correlation has been implemented. The spatial distribution (coherence) of the wind loading is determined from an exponential decay function of the frequency and the geometry of the structure.

Peak results for design are estimated combining mean wind analysis results with root-mean-square buffeting analysis results using the corresponding peak factors. These results are provided in such a way that they can be used for post-processing like any other analysis results produced by *RM2004*.

3. Fundamental aerodynamic characteristics of the bridge deck

The aerodynamic coefficients that are needed for all used cross sections are usually obtained from sectional wind tunnel tests (usually deck section) or from public literature. These aerodynamic coefficients lead to the derivation of curves for drag, lift force and pitching momentum. Aerodynamic coefficients C_D, C_L and C_M depend on the "wind attack angle a". Wind attack angle is an angle between a projection of the wind vector in the element cross plane and a chosen cross plane axis. This axis is axis Z in *Figure 1* below.

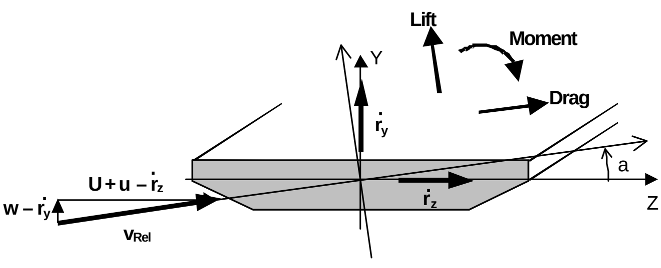


Figure 1: Velocity of the structure and wind fluctuation

$$C_D(a) = \frac{\overline{D(a)}}{\left(\frac{1}{2} r U^2 B\right)}$$

$$C_L(a) = \frac{\overline{L(a)}}{\left(\frac{1}{2} r U^2 B\right)}$$

$$C_M(a) = \frac{\overline{M(a)}}{\left(\frac{1}{2} r U^2 B^2\right)}$$

Equation 1

Drag, lift and moment coefficient are usually given in non-dimensional form. They are normalised with some reference length and area (usually width B and area B²) from the sectional model. Steady state coefficients C_D, C_L and C_M are used to calculate forces on structural elements.

4. Buffeting forces acting on the bridge deck

It is necessary to take into account the possible movement of the structural sections relative to the wind flow in the final expression for the buffeting forces. Only the simple case of the mean wind profile direction vector lying completely parallel to the element local +Z axis will be discussed here. Velocity of the structure in the local Z direction (r_z) and in the local Y direction (r_y) together with a twisting around local X axis (J_x) will change relative to the wind velocity vector and relative to the wind attack angle.

Figure 1 shows the additional wind attack angle α due to the velocity of the structure and wind fluctuation components. Taking into account twisting rotation around local X axis (J_x) total change of the wind attack angle α can be approximated as shown in Equation 3.

$$C_D(a)=C_D+a\cdot C_D'' \ , \qquad C_L(a)=C_L+a\cdot C_L'' \ , \qquad C_M(a)=C_M+a\cdot C_M''$$

Equation 2

$$a \simeq J_x+\frac{w}{U}-\frac{\dot{r}_y}{U}$$ Equation 3

Linearization of the steady state drag, lift and moment coefficient is done around the wind attack angle under the mean wind loading (Equation 3).

Wind buffeting forces are finally separated into static (mean wind), dynamic (wind fluctuation) and aerodynamic terms (aerodynamic damping and stiffness). Static terms are covered in the non-linear static calculation. Dynamic and aerodynamic terms will be analysed in the frequency domain. The final expression for the buffeting forces is shown in Equation 4.

$$\begin{Bmatrix} D \\ L \\ M \end{Bmatrix} = \frac{1}{2}rU^2 \begin{Bmatrix} BC_D \\ VC_L \\ B^2C_M \end{Bmatrix}_{Stat} + rU \begin{Bmatrix} BC_D u+\frac{1}{2}B\left(C_D^{'}-C_L\right)w \\ BC_L u+\frac{1}{2}B\left(C_L^{'}+C_D\right)w \\ B^2C_M u+\frac{1}{2}B^2C_M^{'}w \end{Bmatrix}_{Dyn} - rU \begin{Bmatrix} BC_D\dot{r}_z+\frac{1}{2}B\left(C_D^{'}-C_L\right)\dot{r}_y \\ BC_L\dot{r}_z+\frac{1}{2}B\left(C_L^{'}+C_D\right)\dot{r}_y \\ B^2C_M\dot{r}_z+\frac{1}{2}B^2C_M^{'}\dot{r}_y \end{Bmatrix}_{Damp} + rU^2 \begin{Bmatrix} \frac{1}{2}BC_D^{'}J_x \\ \frac{1}{2}BC_L^{'}J_x \\ \frac{1}{2}B^2C_M^{'}J_x \end{Bmatrix}_{Stiff}$$

Equation 4

5. Mean wind profile - non-linear static calculation

The first step of the analysis is the non-linear static calculation under a given permanent loading and a chosen mean wind profile in a given direction. All structural static nonlinearities can be included by RM2004 in this step. The cable sagging effects, p-delta and large displacement effects, non-linear spring and/or buffer elements are combined with an additional mean wind loading. Even creep and shrinkage effects (additional strains, stress and displacement terms) are included in the analysis from time zero up to the wind event, provided the construction history is known.

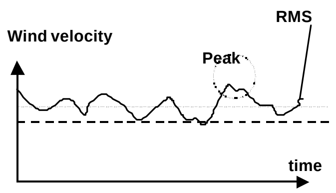


Figure 2: Wind velocity in time domain at one 3D point

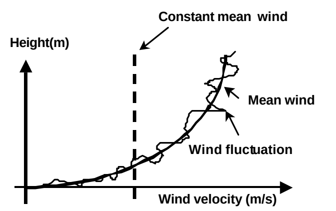


Figure 3: Wind velocity profile over the height

Wind loading has a stochastic nature because wind speeds vary randomly with time. This variation is due to the turbulence of the wind flow. Wind speed at one single 3D point can be separated into the mean wind speed and the fluctuation (turbulence) term as shown in Figure 2.

Mean wind speed varies with the height. Different assumptions for this variation are available, and a usual assumption is some kind of exponential expression: $U(h) = U_{ref} (h/ h_{ref})^{\alpha}$. Coefficient α is an exponent dependant upon roughness of the terrain and varies between 0.15 and 0.4.

Dependency of the steady-state drag, lift and pitching moment coefficients on the wind attack angle are consistently included in this analysis step.

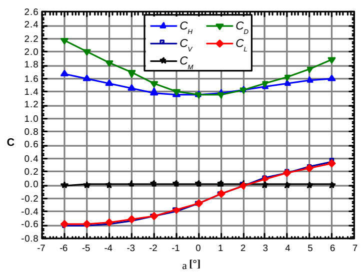


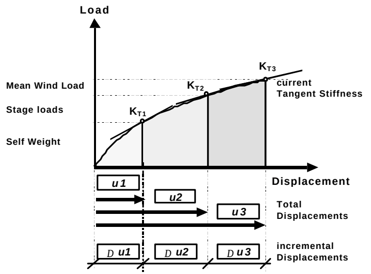
Figure 4: Aerodynamic coefficients of bridge deck for SLS

RM2004 takes interaction of the structure and the mean wind loading into account by recalculating the current position of each structural element due to the mean wind, following each diagram (C_D, C_L, C_M) as shown in Figure 4 (Rach Mieu Bridge).

At each iteration step the mean wind loading is updated and the overall equilibrium is checked with the general Newton-Raphson procedure.

6. Eigenmode Analysis

The next step is a determination of the structural eigenmodes and eigenvalues under current total loading in the displaced structural shape. The state of one fictive structural degree of freedom is shown in Figure 7. This state is prior to the turbulence (fluctuation) wind loading in the numerical analysis. Instead of the linear structural stiffness, the structural "tangent stiffness" under permanent and mean wind loading is used by RM2004, and the classical eigenmode analysis can be performed.



The solution of $|[K_T] - \omega^2 [M]| = 0$ gives eigenmodes and eigenvalues. This step is repeated for each new mean wind direction.

The usage of the structural "tangent stiffness" results in the best possible linearization for the subsequent dynamic analysis steps in the frequency domain.

Figure 5: state of one structural degree of freedom

7. Buffeting analysis in the frequency domain – RMS response

Buffeting loading (fluctuation part) is defined as the unsteady loading of a structure by velocity fluctuation in the oncoming flow. The magnitude of velocity fluctuations is characterised by the non-dimensional quantity termed the turbulence intensity.

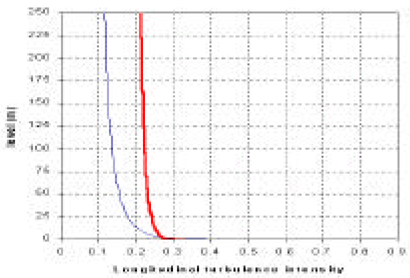


Figure 6

The turbulence intensity component I is defined in Equation 5 below, where s_i is the standard deviation of the fluctuation velocity component in each direction.

$$I_u = \frac{s_u}{U} \qquad I_v = \frac{s_v}{U} \qquad I_w = \frac{s_w}{U}$$

Equation 5

Turbulence intensity generally varies over the height as shown in Figure 8. at the Stonecutters Bridge, Hong Kong

Velocity fluctuation of the incoming flow is usually defined by wind power spectra. Empirical formulae for some well-known spectra implemented in RM2004 are given in Equation 6.

$$S_u(n) = \frac{200 \cdot \frac{z}{U} u_*^2}{\left(1+50\frac{n \cdot z}{U}\right)^{\frac{5}{3}}} \qquad S_v(n) = \frac{15 \cdot \frac{z}{U} u_*^2}{\left(1+9.5\frac{n \cdot z}{U}\right)^{\frac{5}{3}}} \qquad S_w(n) = \frac{3.36 \cdot \frac{z}{U} u_*^2}{\left(1+10\frac{n \cdot z}{U}\right)^{\frac{5}{3}}}$$

Van der Hoven Spectra

$$\frac{f \cdot S_u(f)}{s \cdot u^2} = \frac{c_u^2}{(1+1.5c_u)^{5/3}} \qquad \frac{f \cdot S_v(f)}{s \cdot v^2} = \frac{c_v^2}{(1+1.5c_v)^{5/3}} \qquad \frac{f \cdot S_w(f)}{s \cdot w^2} = \frac{c_w^2}{(1+1.5c_w)^{5/3}}$$

Kaimal Spectra

$$S_u(w) = \frac{4 \cdot k \cdot U_{10}^2 \cdot x^2}{w(1+x^2)^{4/3}} \qquad S_v(w) = \frac{15 \cdot k \cdot U_{10}^2 \cdot y}{w(1+9.5y)^{5/3}} \qquad S_w(w) = \frac{6 \cdot k \cdot U_{10}^2 \cdot y}{w(1+10y)^{5/3}}$$

Davenport Spectra

Equation 6

As a full 3D model is being treated, this fluctuation is given separately by different wind power spectra for longitudinal (u), horizontal (v), and vertical (w) fluctuation component.

Different fluctuation power spectra have been tested and implemented.

Most of the spectra are given in empirical form depending on integral scale lengths, mean wind velocity and frequency.

The cross-spectrum between two points in space is generally complex-valued despite the fact that the single-point spectrum in either of the points is real-valued.

$$|S_{kl}(v)|_{ij} = \sqrt{|S_{kl}(v)|_i |S_{kl}(v)|_j}\, c_{kl}(v),$$

$$c_{kk}(v) = e^{C_{kk}(v)},$$

$$C_{kk}(v) = -\frac{v}{p(\overline{U}_i + \overline{U}_j)} \sqrt{c_x^2 \Delta_x^2 + c_y^2 \Delta_y^2 + c_z^2 \Delta_z^2}$$

Equation 7

The non-zero imaginary part accommodates the fact that, a disturbance occurring in a fluid at one point, will occur at the other point some later time.

The absolute value of the two-point cross-spectrum is generally expressed by means of the root cross-coherence function S as shown in Equation 7, where k, l = u, v or w. Subscripts *i* and *j* refer to the points *i* and *j* in space, respectively. The empirical exponential decay law is widely applied in wind engineering applications, where c_l and Δ_l are the decay coefficient and the separation in the direction of $l = x, y$ or z coordinate.

Multimode buffeting calculation is done in the frequency domain taking aerodynamics damping and stiffness into account. Narrow band analysis as well as broadband analysis with full modal correlation can be done. In bridge engineering, where structural damping is rather small, aerodynamics damping plays an important role as its order of magnitude is the same as that of the structural damping. The result from this calculation step is the RMS envelope result for forces, stresses, displacement, velocity and acceleration.

8. Prediction of the peak buffeting response

The last step in the analysis is a prediction of the expected value of the largest peak. The peak response is taken as the sum of the mean wind response and an RMS buffeting response increased by peak factors K(n).

$$n = \left\{ \frac{\int_0^\infty n^2 S_z(n)\,dn}{\int_0^\infty S_z(n)\,dn} \right\}^{1/2}, \quad K(n) = (2\ln nT)^{1/2} + \frac{0.577}{(2\ln nT)^{1/2}}$$

Equation 8

Mode number	Frequency (Hz)	Aerodynamic damping	Peak factor K(n)
1	0,1484	0,00899	3,1852
2	0,1889	0,01246	3,2568
3	0,1918	0,01115	3,2631
4	0,1975	0,05510	3,2409
5	0,2423	0,04361	3,3150
6	0,3147	0,03409	3,3916
7	0,3507	0,01491	3,4391
8	0,3670	0,00266	3,4596

Table 1: K(n) values for the first 8 modes

Usually it is assumed that the probability distribution of the wind event may be of the Poisson type.

It can be shown that the peak factor K for such a signal depends on its spectral density function and frequency n.

The approximation given in Equation 8 is adequate for use in practical calculations.

Values for displacement peak factor K(n) are usually in the range between 3 and 3.5 as is shown in Table 1 (Stonecutters Bridge, HK).

9. Application examples

Global analysis including wind-buffeting response has been successfully applied to the Rach Mieu Bridge in Vietnam, the Shenzhen Western Corridor and the Stonecutters Bridge in Hong Kong. This experience shows that with a growing span wind events and non-linear effects become important design criteria in bridge engineering.

The Rach Mieu Cable Stayed Bridge (Project Engineers: Transport Engineering Design Inc., Hanoi) with a span arrangement of 117 : 270 : 117 m comprises a prestressed concrete deck girder with reinforced concrete pylons and substructure

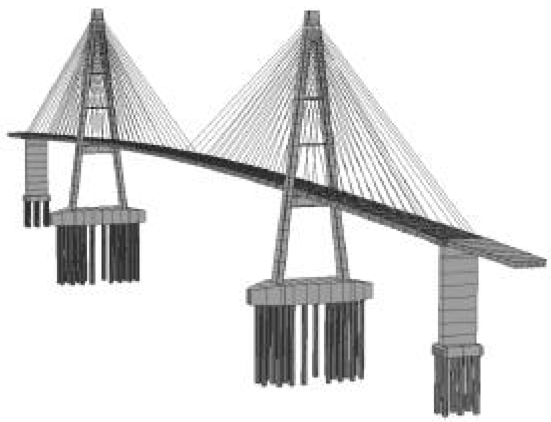


Figure 7: Structural RM2004 model, Rach Mieu Bridge, Vietnam.

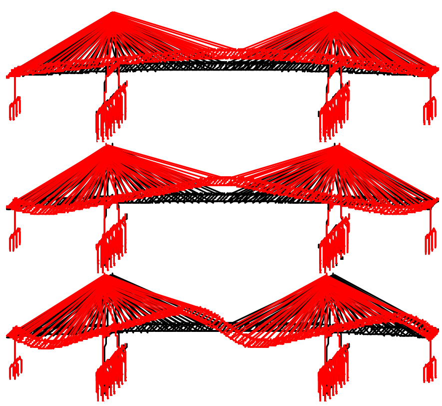


Figure 8: Eigenmodes mainly induced in buffeting response

A cable stayed bridge with a main span of 210 m is on the Hong-Kong side of the Shenzhen Western Corridor (Project Engineers: Ove-Arup, Hong-Kong).

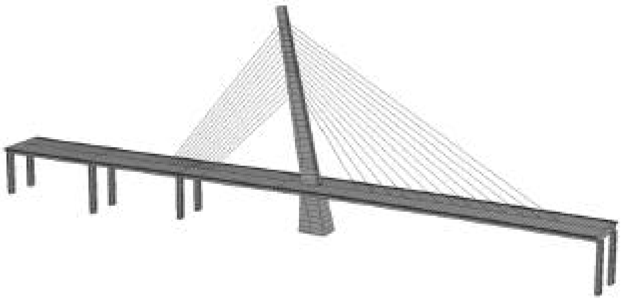


Figure 9: Structural RM2004 model, Shenzhen Western Corridor

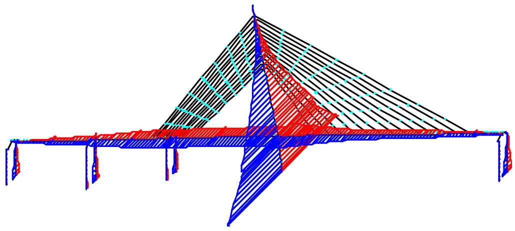


Figure 10: Lateral bending moment

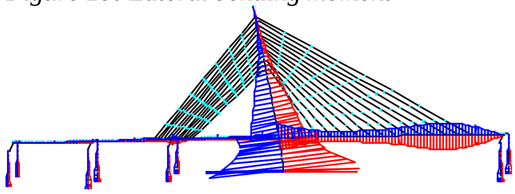


Figure 11: Vertical bending moment

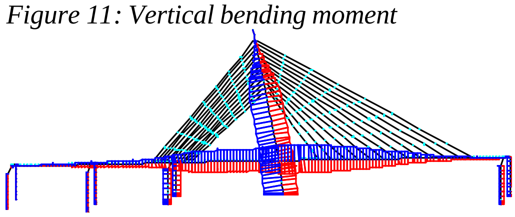
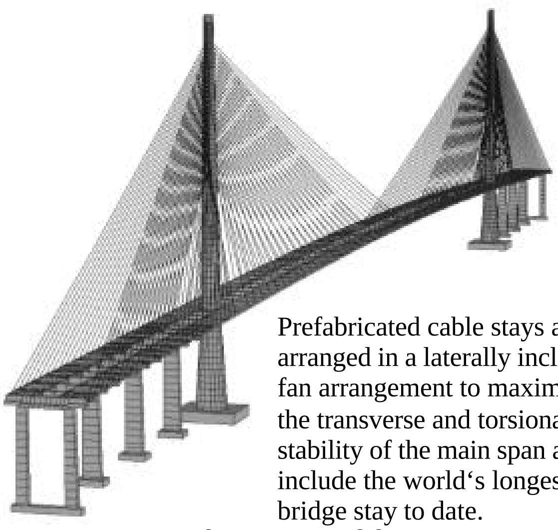


Figure 12: Normal force

The steel girder is supported by an inclined pylon 158m high. Wind buffeting responses are shown in Figure 10 to Figure 12.

The Stonecutters bridge (Project Engineers: Ove-Arup, Hong-Kong) is a cable-stayed bridge with a main span of 1018m, side spans of 298m, and a towers height of 290m. The deck to the main span is a twin girder steel deck, whilst the side spans are of concrete, which are to be built in advance of cable stay erection to counterbalance and stabilise the slender lightweight main span deck.



Prefabricated cable stays are arranged in a laterally inclined fan arrangement to maximise the transverse and torsional stability of the main span and include the world's longest bridge stay to date.

Figure 13: Structural RM2004 model, Stonecutters Bridge, HK

Wind buffeting responses are shown in Figure 14 to Figure 16.

Figure 14: Tower bending moment

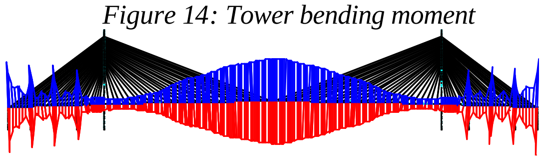


Figure 15: Deck bending moment

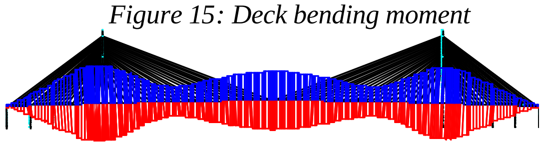


Figure 16:Deck normal force

10. References

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