Abstract

Wherever built, suspension bridges attract public attention due to their size and conspicuousness. However, the long spans combined with extraordinary slenderness yield outstanding challenges.

First of all, in any case the slenderness and kinematical conditions of these structures bring about large displacements due to the permanent loads. Therefore, the shape of the bridge is a non-linear function of the loading, deviating to a great extent from the hypothetical “stress-less” shape. The form finding process is a complicated iterative process if done in the conventional way. As an alternative, the Additional Constraint Method has been provided in the program RM2006 in order to find and optimize the shape of the suspension cables and the hangers.

A further great challenge is the simulation of the erection process. Further on, un-symmetric loading due to traffic causes large displacements and requires non-linear traffic analyses. Last but not least, a major engineering challenge of long suspension bridges is their susceptibility to wind induced vibrations.

The Hardanger Bridge project is used as a descriptive example for an integrative procedure including form finding, simulation of the erection process, and detailed analysis with considering geometric non-linearity and dynamic impacts like wind induced vibrations.

Keywords: Suspension bridge, form finding process, fabrication shape, erection procedure, wind impact, buffeting analysis, Hardanger Bridge

1. Introduction

Although the construction of modern type suspension bridges dates back more than 120 years, bridges of this type have still a special fascination due to their architectonical elegance combined with a touch of lightness predestining them to become landmarks wherever built. For bridge engineers this fascination is also based on the size of these structures, with allowable spans being longer than for any other bridge type. The long spans combined with extraordinary slenderness yield outstanding challenges for any bridge designer.

Non-linear behaviour: First of all, the slenderness and kinematical conditions of these structures yield in any case large displacements due to the permanent loads. Therefore, the shape of the bridge is a non-linear function of the loading, deviating to a great extent from the hypothetical “stress-less” shape. Due to the high non-linearity of the problem, the usual straight-forward design approach for conventional structures – i.e. using the desired design shape in the analysis and compensating the deformations in the erection process by applying appropriate pre-camber values – is not suitable anymore. Therefore, a complicated form-finding process with taking into account geometrical nonlinearities is required. This process is described in detail in the next section.

In addition to the geometric non-linearity, various other non-linear mechanisms generally occur. They require on the one hand the use of special element type, and on the other hand a global
solution concept allowing for dealing with the different non-linearity types in a comprehensive approach. Typical problems requiring special element types are for instance

- Cable sagging, which requires special cable elements. These elements describe the non-linear stiffness due to sagging. A special element type has been developed. It guarantees stable behaviour even with very large displacements and any imperfections of the stress-less geometry as shown in Figure 1.

![Fig. 1 Special cable element for calculating large displacements](image)

- Fixing the suspension cable at the top of the pylon generally induces high and illegal bending moments in the pylon. Therefore, a saddle is usually arranged on the top of the pylon allowing for slipping of the suspension cable in the erection process. This connection is modelled by friction elements, where the transmitted horizontal forces are a function of the vertical redirection force.

- The lateral gaps between the main girder and the pylon legs are modelled with special gap elements, which confine the free lateral displacements of the main girder to a certain amount.

- Eccentric hinge elements allow for simulating any distance pieces required in the construction stage for keeping the individual segments in position without inducing illegal constraints.

With respect to the overall solution algorithm for the non-linear equation system, a Newton-Raphson iteration scheme as shown in Figure 2 is used. This iteration scheme turned out to guarantee numerical stability in all situations, even in highly non-linear dynamic analyses.

![Fig. 2 Newton-Raphson scheme for non-linear static and dynamic calculation](image)
2. Form Finding Process

The *form finding process* – i.e. determining the theoretical “stress-less” state of the structural components – is a backward iteration process, being rather complicated and time consuming if done in the conventional way. As an alternative to the conventional approximate hand calculation, the *Additional Constraint Method* has been provided in the program RM2006 in order to find and optimize the shape of the suspension cables and the hangers.

![Fig. 3 Schematic view of a suspension bridge](image)

The final shape of a suspended cable depends on the loading of the cable and on the cable tensioning force. In suspension bridges, transverse loads on the suspension cable are caused by the self-weight and loading of the super-structure, which is transmitted to the suspension cable via hangers. The normal force in the suspension cable is anchored at the anchor blocks.

![Fig. 4 Constraints for determining the shape of the suspension cable](image)

In the design process, the required stress-free cable lengths yielding the intended design shape under a certain loading, have to be determined. Either the cable normal force or the cable sag is of a given design value. The *Additional Constraint Method* implemented in RM2006 can be employed to find the shape of the suspension cable and the hangers. Some aspects of using this method in the design process of the Hardanger Bridge going to be built in Norway are shown in chapter 6.
Consider the small example shown in Figure 4(a). In this small example, a suspension cable and two hangers support a girder symmetrically. The structure is initially modelled with an approximate geometry and pre-stressing force. The assumed design task for this example will be to determine necessary corrections to achieve a cable geometry for a horizontal girder, vertical hangers and a given horizontal force $H$ at the ends of the suspension cable under self-weight. Two possibilities exist to model this problem with using the \textit{Additional Constraint Method}:

a) The original structural system can be used and \textit{Additional Constraints} can be introduced in the girder and at the top of the hangers, to set the respective displacements zero. Additionally, with using another \textit{Additional Constraint}, the horizontal force $H$ can be set to a given value. Normal forces in the cable portions are used as variable \textit{degrees of freedom} to achieve the design goals specified as \textit{Additional Constraints} (Figure 4b). This variant is numerically more expensive and less stable, but the procedure can be performed on the original structural model without modifications.

b) Spring elements are added at the locations of the \textit{Additional Constraints}, and the reaction forces in these elements as well as the horizontal force $H$ are set to zero. Again, the normal forces in the cable portions will be used as variable \textit{DOFs} to achieve the design goals (Figure 4c). This variant requires some changes in the structural model since additional spring elements must be introduced. However, non-linearities in the equation system describing the design target (i.e. additional constraints) are smaller and the solution is more stable and faster.

3. \textbf{Calculation Procedure for Simulating the Erection Process}

A further great challenge is the simulation of the \textit{erection process}. In the \textit{Hardanger} project shown later, the erection process has been designed in accordance with the experiences gained in previous Norwegian suspension bridges. After building the pylons in the traditional way, the bridge deck is constructed by elevating the bridge deck sections from a floating barge on the fjord. In order to obtain the “correct” geometry of the bridge deck, all deck sections are continuously elevated and connected to the hangers. A temporary connection between the deck sections will be installed.

When all deck sections are mounted, the welding procedure can start.

It is clear, that the procedure of connecting the different girder segments one after the other to the respective hangers causes continuously considerable changes of the sagging curve of the suspension cables in accordance with the weight of the actually mounted segments. This leads to high up and down movements of the deck segments during erection. Preliminary hinged connections with some spacing have to be applied between the segments in order to kinematically allow these movements without inducing impermissible constraints, and with avoiding that the segments bang together uncontrollably.

The final shape is approximately reached when all when all segments are mounted. Controlled removal of the distance pieces closes the gaps between the segments and allows for starting the welding process. The final shape is definitely reached, when all segments are welded. Continuous adaptation of the mathematical model is recommended still in the erection phase. Any deviations of the actual behaviour from the predicted one can be detected in an early stage and the required compensation measures can easily be fixed by performing appropriate erection control calculations.

4. \textbf{Consideration of Traffic Loading}

Most commonly used – and the first step in the more sophisticated procedure – is considering full geometric non-linearity for all permanent loads, and assuming linear behaviour for the traffic loading with using the tangent matrix of the system calculated after the dead load has been applied. However, suspension bridges mostly require a fully non-linear calculation also for the traffic load.

This is done in RM by automatically creating the appropriate load sets describing the load train in the worst position. These load sets can be included in the total load case allowing for performing a geometrically nonlinear calculation. Using the „Accumulate permanent load“ option in the calculation process allows in fact using these load sets also in incremental load cases.

The consistent RM procedure for performing a nonlinear traffic analysis is

1. Non-linear solution for the permanent loading (module CALC)
2. Calculation of the respective Tangent Stiffness Matrix (module CALC)
3. Calculation of influence lines (module INFL)

4. Determination of the critical traffic load position for every component of the result vector (displacements, internal forces) (module LIVESET) as shown in Figure 5 for a typical internal force vector \(N\), \(Q_y\) and \(M_z\).

5. Generation of respective load cases (module LIVESET)

6. Non-linear calculation of these load cases and replacing the respective result values in the envelope (module CALC).

![Fig. 5 Influence lines with worst position of complicated traffic loading](image)

### 5. Wind induced vibrations

Last but not least, a major engineering challenge of long suspension bridges is their susceptibility to wind induced vibrations. Referring to wind impact, long-span bridges require sophisticated wind buffeting analyses with considering both, the aero-elastic behaviour of the structure and the wind loading correlation.

The first step in the wind design process is to consider the additional steady state load due to mean wind forces. These loads are usually derived from the design wind speed by multiplying the respective dynamic pressure with the dimensionless steady-state drag, lift, and moment coefficients \(C_{D}(\alpha)\), \(C_{L}(\alpha)\), and \(C_{M}(\alpha)\), respectively, where \(\alpha\) denotes the wind incident angle. Strictly speaking, the coefficients are dependent on the Reynolds number and turbulence properties of the airflow. However these effects are in general neglected.

Dynamic excitations are mainly caused by two mechanisms. First, the mean wind is superimposed by statistical fluctuations, which lead to buffeting forces. These fluctuations are called “turbulence intensities” which describe the deviations of the actual wind vector from the steady state. The power spectrum describes the energy content of these turbulences in dependency from the frequency. Coherence data describe the spatial distribution of these turbulences. All these parameters are stored in the database of the mathematical model and allow for performing a sophisticated buffeting analysis with considering the respective above-mentioned steady state coefficients and their derivates with respect to the attack angle.

The coupling of girder vibrations (vertical and torsional) with the mean flow is described by the flutter equations. Classical flutter comprises only the coupling of lift and moment terms with the flow. However, an expansion to including drag forces is generally required for suspension bridges. In this case, lift forces as well as drag forces are coupled with the respective moment. Each of the three aero-elastic forces is determined by four coefficients, the flutter derivatives.

A phenomenon closely related to flutter is the lock-in of the vortex shedding frequency. Normally, the shedding frequency increases linearly with wind velocity. In lock-in regions, the natural frequency influences the vortex shedding behaviour, i.e. the natural frequency of the structure is imposed to the vortex shedding frequency, independently of the velocity. An example is shown in Figure 6. We see, that the variation of the lift coefficient (which corresponds to the vortex shedding frequency) occurs with the frequency of the vibration of the girder, although the theoretical vortex shedding frequency related to a static cross-section is different.

Once the steady state coefficients and flutter derivatives are known, the vortex induced vibration analysis can be performed by methods presented e.g. in [2]. However, a closed computer aided
design process is interrupted if these coefficients must be determined externally, for example by wind tunnel tests. In order to overcome this handicap, a CFD module has been implemented in the program RM2006, which is able to calculate steady state coefficients as well as flutter derivatives within the analysis process.

Fig. 6 Lift coefficient (blue) and natural vibration of the cross-section (brown)

Together with the above-mentioned possibility of entering the parameters of the oncoming wind as parameters of the database object “wind profile”, this CFD calculation of the aerodynamic coefficients and their derivatives enables – with respect to dealing wind related problems – a closed computer aided design circle within RM2006.

6. Example

The Hardanger Bridge to be built in western Norway is used as an example for an integrated computer aided design procedure of a major suspension bridge. This bridge will cross the Hardanger fjord with a main span of 1310 m, which is no. 7 of the current ranking of worlds longest suspension bridges. The Norwegian road authority Statens Vegvesen in close collaboration with TDV at Graz, Austria, the supplier of the software package RM, performs the design work.

The mathematical model of the bridge and the construction schedule is continuously attuned, further developed and improved by the 2 parties, i.e. all documentation and the currently valid database of the model is stored in an internet E-room accessible to the engineers involved in the different investigation tasks.

Fig. 7 Structural model of the Hardanger Bridge

The mathematical model of the bridge is primarily based on the intended design shape developed by Statens Vegvesen in previous feasibility studies during the run-up time to the actual design phase. Due to the steep slopes into the fjord, the pylons have been placed in the shallow water very near to the shore. Therefore, as shown in Figure 7, the bridge has negligible side spans, and the main cables are anchored in separate anchor blocks on land. The main design shape parameters are the distance
and height of the pylons, the position of the anchor blocks and the cross-sections of the different members such as pylon legs and main girder.

**Form finding:** The form-finding process with using the *AddCon* method was based on the provisional assumption of a straight girder. Three sets of constraints were applied to get the required cable lengths yielding the desired shape under permanent loading. These constraints are

- vertical displacements of the bottom points of the hangers must be zero
- horizontal displacements of the top points of the hangers must be zero
- the main cable sagging must get the intended value, i.e. vertical displacement of the central hanger top point must be zero

The respective variables for maintaining these constraints are the required length adjustments of the hangers (H1, H2, H3, …), the main cable sections (C1, C2, C3, …) and the back spans (C2009, …) as shown in Figure 8. In addition to this initial form finding process, for additionally imposing the required pre-camber of the main girder, optimisation procedures for the respective bending moments were adopted.

**Wind:** Investigations of the behaviour of the bridge under heavy wind have been found necessary for both, the construction stage, where the different segments of the superstructure have not yet been connected, and for the final stage with and without traffic. The respective analyses include investigations of the aerodynamic behaviour of the flow around the different structural members.

Figure 9 shows the flow around the superstructure cross-section calculated with the CFD module of the program RM. The different colours indicate the deviation of the local direction-independent wind velocity from the mean wind. This calculation directly gives the relevant aerodynamic coefficients for the further static and dynamic wind analyses of the structure. Independently, extensive wind tunnel tests have been performed. Comprehensive comparisons showed a generally good coincidence of the calculated coefficients with the measurement results.
A similar investigation has been performed for the flow around the pylon consisting of two legs with varying distance. In this case, the variation of the relevant parameters with varying wind directions was of a special interest. The results of these investigations are shown in detail in [1].

Fig. 9 Airflow around the superstructure cross-section of the Hardanger Bridge

Based on the results of these preparatory investigations, sophisticated static and dynamic wind analyses (vortex shedding, buffeting analyses) could be performed straight forward for the different construction states and for the final state. The buffeting analyses were based on the wind specifications of the Norwegian design code and information of the meteorological authorities, entered in the model database as so-called “wind profiles”. These wind profiles are parameter sets describing the magnitude as well as the time and space distribution of the oncoming design wind.

7. Summary

Suspension bridges yield special requirements for the design process. One of the tasks is the form finding process allowing for designing the proper geometry of suspension cables, hangers and superstructure segments in order to enable the assembly process and to get the intended design shape under permanent loading. A further challenge is the design of the optimal erection process. Geometric non-linearity has to be taken into account in all analysis tasks, including traffic and wind investigations.

Due to the slenderness of the respective structures, special attention has to be turned on wind-induced vibrations, with static and dynamic wind excitations to be investigated. An integral solution of the different topics related to wind impact is shown in the paper. As an example, the integrated investigations performed in the Hardanger Bridge project have been presented. This integrated approach considerably eases and enhances the overall design process.

References